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which is the circle, doubly counted, resulting from making S_1 and S_2 poin circles in both figures. As in the preceding case, the locus of P in Fig. 1 is also a circle. The general locus is a bicircular quartic, hence its inverse is also a bicircular quartic, thus determining the locus of P (Fig. 1) which constitutes the solution of the given problem.

NOTE. No solutions of 374 and 376 have yet been received. We shall be pleased to have our contributors take up these two problems for solution. ED. F.

CALCULUS.

300. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Solve the differential equation obtaining the complete primitive,

$$(x^2+x^2y+2xy-y^2-y^3)dx+(y^2+xy^2+2xy-x^2-x^3)dy=0.$$

Solution by V. M. SPUNAR, Chicago, Ill., and the PROPOSER.

If F is an integrating factor of the differential equation

$$Mdx+Ndy=0,$$

we have the relation

$$\frac{F'(v)}{F(v)}=\frac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{N\dfrac{\partial v}{\partial x}-M\dfrac{\partial v}{\partial y}}=\frac{4(x-y)(x+y+1)}{N\dfrac{\partial v}{\partial x}-M\dfrac{\partial v}{\partial y}}.$$

On account of the symmetry of M and N, it is evident that v is symmetrical in x and y. Trying $v=x+y$, we have

$$\frac{F'(v)}{F(v)}=-\frac{4(x+y+1)}{(x+y)^2+2(x+y)}=-\frac{4(v+1)}{v^2+2v}.$$

Integrating, we have,

$$F=\frac{1}{v^2(v+2)^2}=\frac{1}{(x+y)^2(x+y+2)^2}.$$

Trying, also, $v=(1+x)(1+y)$, we have,

$$\frac{F'(v)}{F(v)}=-\frac{2}{x+xy+y}=-\frac{2}{v-1}.$$

Integrating,

$$F=\frac{1}{(v-1)^2}=\frac{1}{(x+xy+y)^2}.$$

If we have two integrating factors, their ratio, put equal to a constant, is the complete primitive.

Therefore, the complete primitive is

$$\frac{(x+y)^2(x+y+2)^2}{(x+xy+y)^2}=c,$$

or, extracting the square root,

$$\frac{(x+y)\,(x+y+2)}{x+xy+y}=c_1,$$

or, in the form,

$$\frac{x^2+y^2}{x+x\,y+y}=c_2.$$

301. Proposed by C. N. SCHMALL, New York City.

Show that the volume of the surface,

$$\left(\frac{x}{a}\right)^{\frac{2}{5}}+\left(\frac{y}{b}\right)^{\frac{2}{5}}+\left(\frac{z}{c}\right)^{\frac{2}{5}}=1,\ \text{is}\ \frac{100\,\pi\,abc}{3.5.7.11.13}.$$

I. Solution by FRANCIS RUST, C. E., Pittsburg, Pa.

Volume is special case of Derichlet's special formula:

$$\int\int\int\dots dx\,dy\,dz\dots x^{l-1}.y^{m-1}.z^{n-1}\dots$$

$$=\frac{a^l.b^m.c^n\dots}{p.q.r\dots}\cdot\frac{\Gamma\left(\frac{l}{p}\right).\,\Gamma\left(\frac{m}{q}\right).\,\Gamma\left(\frac{n}{r}\right)\dots\dots}{\Gamma\left(1+\frac{l}{p}+\frac{m}{q}+\frac{n}{r}+\dots\right)},$$

provided, $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r+\dots=1$, and the integration to extend over all *positive* values of the variables.

In this case,

$$V=\int\int\int dx\,dy\,dz=\frac{abc}{p.q.r}\cdot\frac{\Gamma\left[\frac{1}{p}\right].\Gamma\left[\frac{1}{q}\right].\Gamma\left[\frac{1}{r}\right]}{\Gamma\left[1+\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right]}$$

with $p=q=r=\frac{2}{5}$. The formula yields $\frac{1}{8}$ part of the total volume, which consequently is